

Modelyze: Embedding Equation-Based DSLs

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Agenda

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Part I

Modelyze Overview



Part II

Formal Semantics

$$\frac{\begin{array}{l} \Gamma \vdash_L e_1 \rightsquigarrow e'_1 : \langle \tau_{11} \rightarrow \tau_{12} \rangle \\ \Gamma \vdash_L e_2 \rightsquigarrow e'_2 : \tau_2 \\ [e'_2 : \tau_2] = e''_2 \\ \langle \tau_{11} \rangle \sim [\tau_2] \end{array}}{\Gamma \vdash_L e_1 e_2 \rightsquigarrow e'_1 @ e''_2 : \langle \tau_{12} \rangle} \text{ (L-APP5)}$$

Part III

Modelyze Demo



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Part II

Modelyze Overview



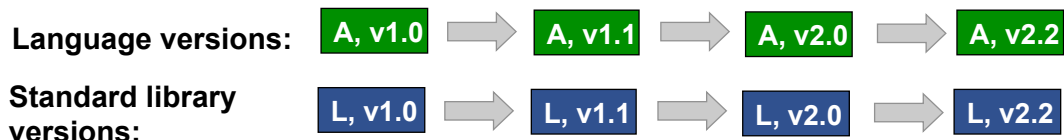
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Problem: Expressiveness and Analyzability

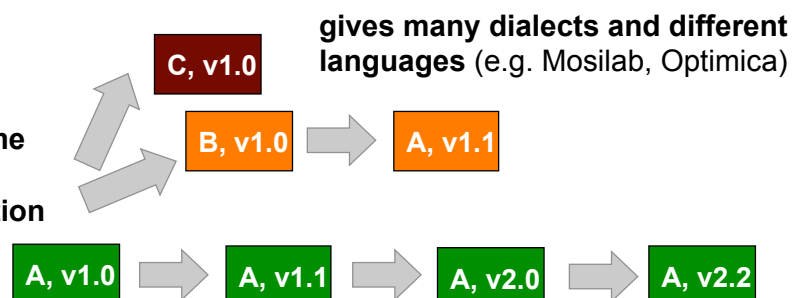
Cannot express all modeling or analysis needs.
Limited to what the modeling language can provide.



Modelica: A new language definition approximately every second year

Uses

- Simulation
- Optimization
- Code generation for real-time
- Model export
- Grey-box system identification etc.



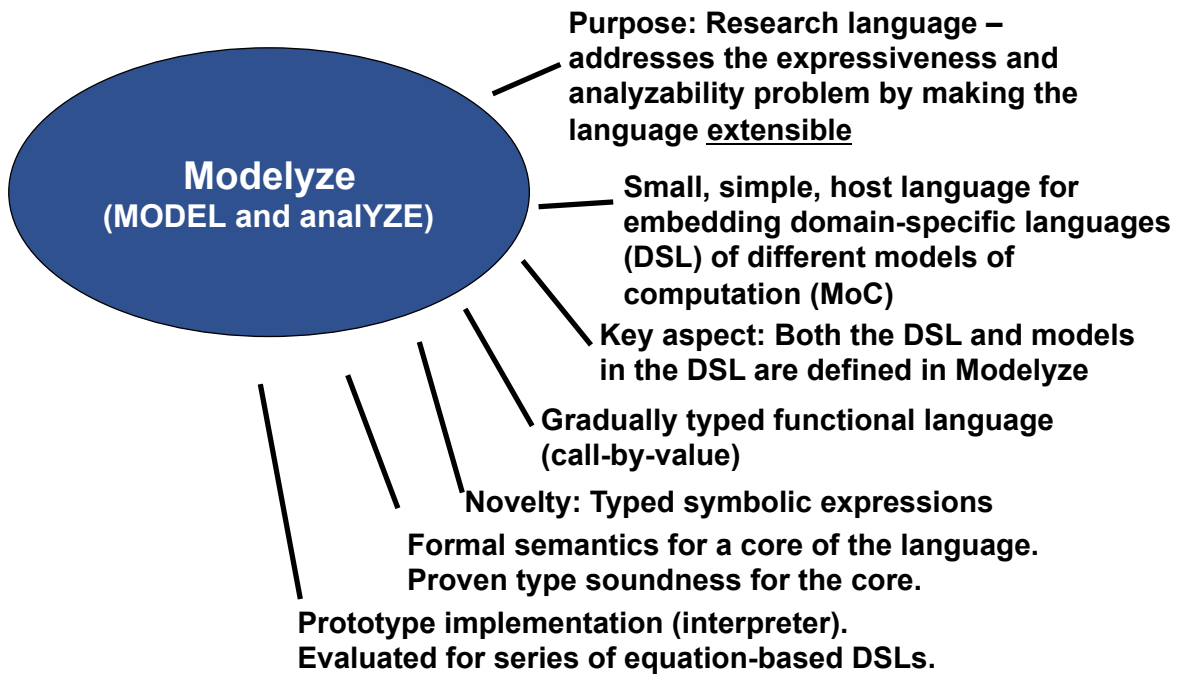
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What is Modelyze?

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Experimental DSLs

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Extensible DSLs for physical modeling

Differential-Algebraic Equations (DAE)

ModelyzeDAE

Acausal connections (Electrical and Mechanical domain)

ModelyzeEOO

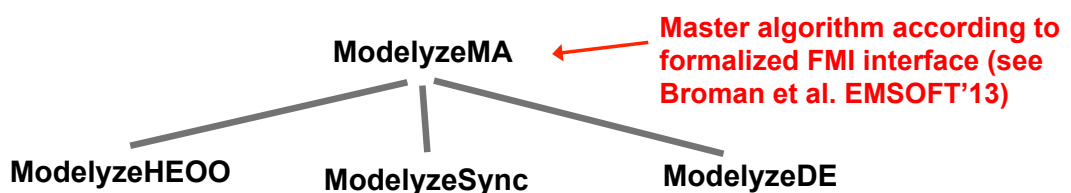
HybridCharts (DAE with modes)


ModelyzeHC

EOO + HC = HEOO

ModelyzeHEOO

Ongoing work on combining heterogeneous DSLs for CPS



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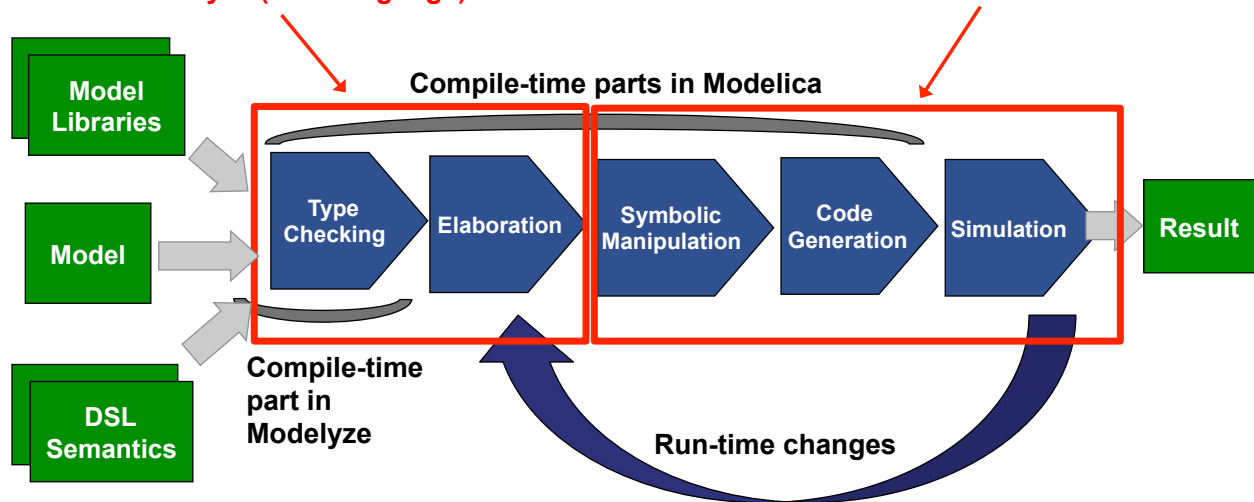
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Overview of the Compilation and Simulation Process

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Type checking and collapsing the instance hierarchy come “for free”.
Part of Modelyze (host language)

Run-time semantics described in Modelyze libraries (meta-programming)



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Related Work

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Implementing DSLs

Compiler construction

- JastAdd (Ekman & Hedin, 2007)
- MetaModelica (Pop & Fritzson, 2006)

Preprocessing and template metaprogramming

- C++ Templates (Veldhuizen, 1995)
- Template Haskell (Sheard & Peyton Jones, 2002)
- Stratego/XP (Bravenboer et al., 2008)

Embedded DSLs

- Haskell DSEs, e.g., Fran (Ellito & Hudak, 1997), Lava (Bjesse et al. 1998), and Paradise (Augustsson, 2008)
- FHM (Nilsson et al., 2003)
- ForSyDe (Sander & Jantsch, 2004)
- Pure embedding (Higher-order functions, polymorphism, lazy evaluation, type classes) (Hudak, 1998)

Combining Dynamic and Static Typing

- Gradual Typing (Siek & Taha, 2007)
- Soft Typing (Cartwright & Fagan, 1991)
- Dynamic type with typecase (Abadi et al., 1991)
- Typed Scheme, Racket (Tobin-Hochstadt, Felleisen, 2008)
- Thorn, like types (Wrigstad et al., 2010)

Representing Code and Data type

- Dynamic languages LISP, Mathematica
- MetaML <T> (Taha & Sheard, 2000)
- GADT (Peyton Jones et al., 2006; Xi et al., 2003; Cheney & Ralf, 2003)
- Open Data types (Löh & Hinze, 2006)
- Pattern Calculus (Jay, 2009)
- Syntactic library (Axelsson, 2012)



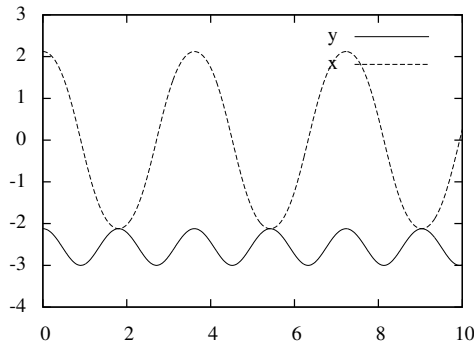
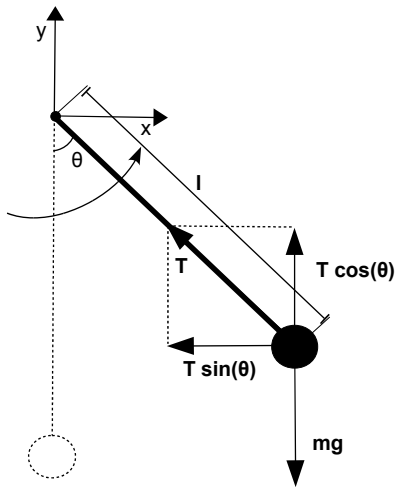
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Pendulum Example

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Differential-Algebraic equations

Algebraic constraint

Initial values

$$\begin{aligned} -T \cdot \frac{x}{l} &= m\ddot{x} & x(0) &= l \sin(\theta_s) \\ -T \cdot \frac{y}{l} - mg &= m\ddot{y} & y(0) &= -l \cos(\theta_s) \\ x^2 + y^2 &= l^2 \end{aligned}$$



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Declarative Mathematical Model

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Using function abstraction to define the model

Unknowns are given types but not bound to values

Equations and initial values are defined declaratively, just as the mathematical equations

```
def Pendulum(m:Real, l:Real, angle:Real) = {
  def x, y, T:Real;
  init x (l*sin(angle));
  init y (-l*cos(angle));

  -T*x/l = m*x'';
  -T*y/l - m*g = m*y'';
  x^2. + y^2. = l^2.;
}
```

$$\begin{aligned} -T \cdot \frac{x}{l} &= m\ddot{x} & x(0) &= l \sin(\theta_s) \\ -T \cdot \frac{y}{l} - mg &= m\ddot{y} & y(0) &= -l \cos(\theta_s) \\ x^2 + y^2 &= l^2 \end{aligned}$$



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Declarative Mathematical Model

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Which parts are part of the host language (Modelyze)?

Unknowns are internally represented as typed symbols


Fresh (unique) symbol $S:\mathcal{T}$ Tagged with a type

$\langle \mathcal{T} \rangle$ Symbolic type

```
def Pendulum(m:Real, l:Real, angle:Real) = {
  def x,y,T:Real;
  init x (l*sin(angle));
  init y (-l*cos(angle));

  -T*x/l = m*x'';
  -T*y/l - m*g = m*y'';
  x^2. + y^2. = l^2.;
}
```

Variable x is bound to fresh a symbol of type $\langle \text{Real} \rangle$

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Release the user from annotation burden

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Symbols cannot be bound to values, so x^2 would crash at runtime

Use quasi-quoting to mix symbolic expressions and program code?


Using MetaML syntax $\langle \rangle$ for quotation and \sim for anti-quoting (escape)

```
<~x^2. + ~y^2. = ~((fun t -> <t>)l^2.)>;
```

Heavy annotation burden for the end-user

```
def Pendulum(m:Real, l:Real, angle:Real) = {
  def x,y,T:Real;
  init x (l*sin(angle));
  init y (-l*cos(angle));

  -T*x/l = m*x'';
  -T*y/l - m*g = m*y'';
  x^2. + y^2. = l^2.;
}
```

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Symbol Lifting Analysis (SLA)

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Symbol Lifting Analysis (SLA): During type checking, lift expressions that cannot be safely evaluated at runtime into symbolic expressions (data).

$$\Gamma \vdash_L e \rightsquigarrow e' : \tau$$

Rewritten to prefix curried form

`((/) x) 1`

where

`(/):Real-> Real -> Real`

`x:<Real>`

`l:Real`

```
def Pendulum(m:Real, l:Real, angle:Real) = {
  def x, y, T:Real;
  init x (l*sin(angle));
  init y (-l*cos(angle));

  -T*x/l = m*x'';
  -T*y/l - m*g = m*y'';
  x^2. + y^2. = l^2.;
}
```

`((lift(/):Real-Real->Real) @ x) @ (lift l:Real)`

Resulting type

`<Real>`

Division cannot be performed, lift expression to type `<Real-> Real -> Real`.

Term `lift e:T` wraps `e` and results in type `<T>`

Term `e1@e2` is a symbolic application, represented as a tuple.



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Pattern Matching on Symbolic Expressions

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Dynamic symbolic type `<?>`

Query for all unknowns in a model instance

Accumulator Sets of symbolic type `<Real>`

```
def getUnknowns(exp:<?>, acc:(Set <Real>)) -> (Set <Real>) = {
  match exp with
  | e1 e2 -> getUnknowns(e2, getUnknowns(e1, acc))
  | sym:Real -> Set.add exp acc
  | _ -> acc
}
```

Uniform data structure, no boilerplate code (matching on symbolic applications)

Match all symbols of type `<Real>` i.e., unknowns in the model.

`getUnknowns(Pendulum(5, 3, 45*pi/180), Set.empty)`

Returns a set with 3 symbols (representing `x`, `y`, and `T`).



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Static Error Checking at the DSL Level

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Syntactically correct
model (host syntax)

Static type error instead of dynamic error
during translation/pattern matching.

```
def ModifiedPendulum(m:Real,l:Real,angle:Real) = {
  def x,y,T:Real;
  init x (l*sin(angle));
  init y; //Error: Missing initial value

  -T*x/l = m*x'';
  -T*y/l - m*g = m*y'';
  x^2. + y^2. = l^2.;
}
```

Quite intuitive error messages at the DSL level.

```
modifiedpendulum.moz 4:10-4:10 error: Missing argument
of type 'Real'.
```

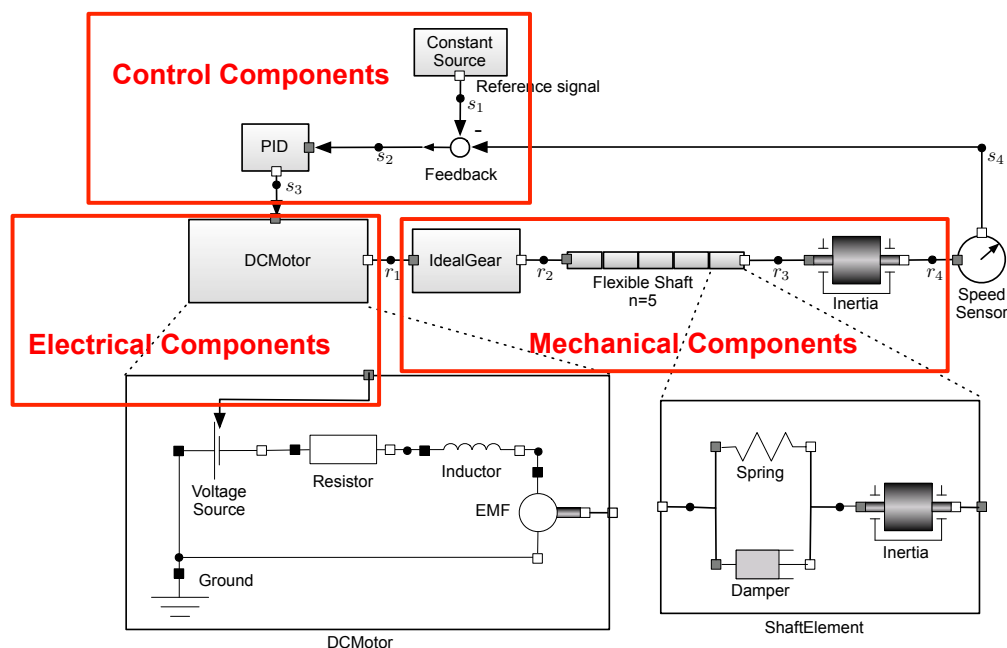
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Mechatronic Control Example (ModelyzeEEO)

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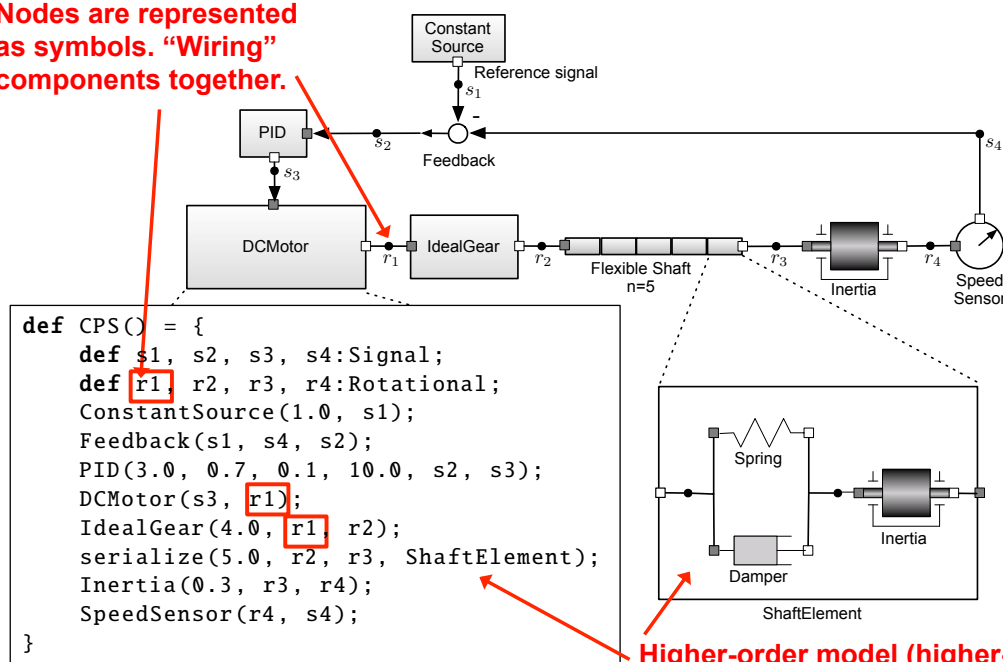
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Mechatronic Control Example

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Nodes are represented as symbols. "Wiring" components together.



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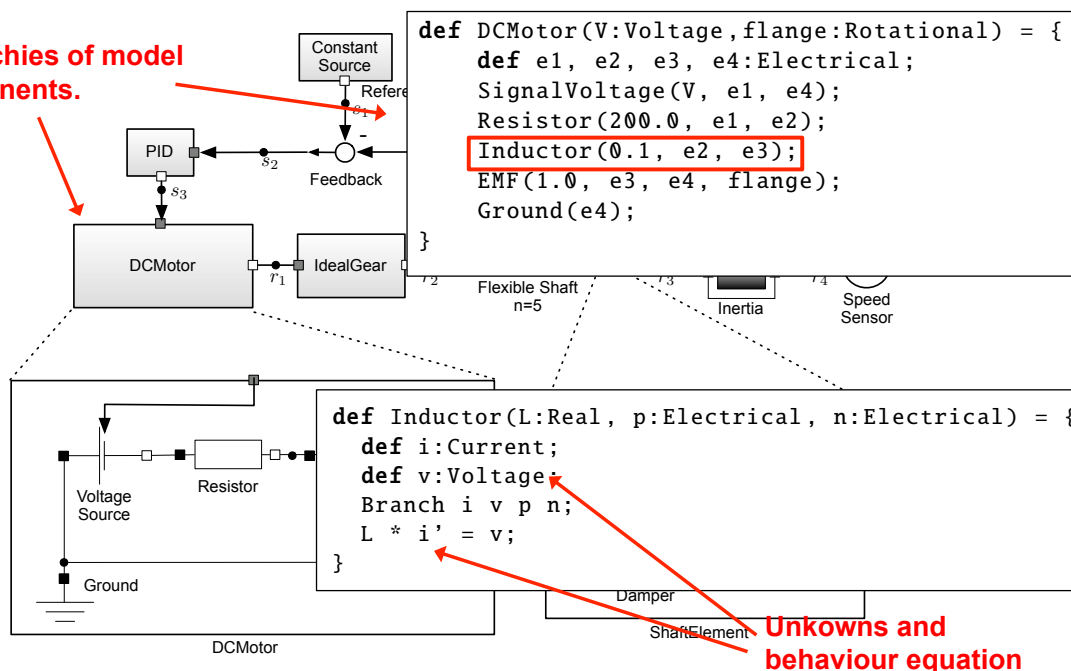
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Mechatronic Control Example

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Hierarchies of model components.



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Part II Formal Semantics

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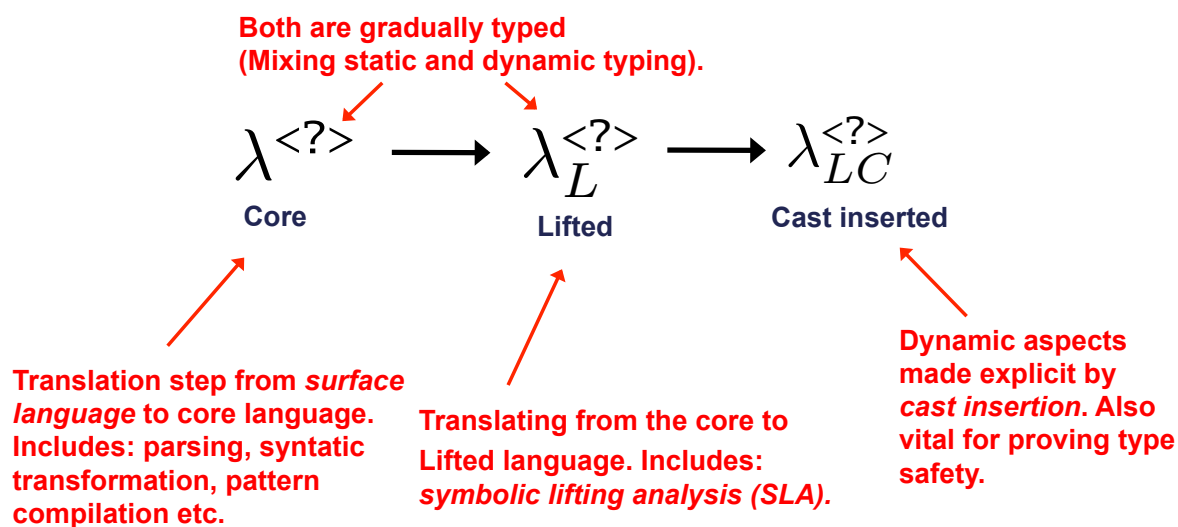


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Intermediate Languages

To enable formalization and proving type soundness, we define three intermediate languages.



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Abstract Syntax

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Core

 $\lambda\langle?\rangle$

Ground Types	$\gamma \in \mathbb{G}$	
Symbolic Data Types	$D \in \mathbb{D}$	
Types	$\tau ::= \gamma \mid \tau \rightarrow \tau \mid ? \mid \langle \tau \rangle \mid D$	
Variables	$x, y \in \mathbb{X}$	
Symbols	$s \in \mathbb{S}$	
Constants	$c \in \mathbb{C}$	
Expressions	$e ::= x \mid \lambda x:\tau.e \mid e e \mid c \mid \mathbf{error} \mid \nu(\tau) \mid \mathbf{case}(e, p, e, e)$	
Patterns	$p ::= \mathbf{sym}:\tau \mid x @ x \mid \mathbf{lift} x:\tau$	

Ranges over ground types (Int, Real, etc.).
Function and dynamic types.
Symbolic type.
Symbolic data type (e.g., the equation above).
Standard expressions (var, lambda, application, constant, error).
Case expression for eliminating symbolic data. Three forms of patterns.
"new" creates a new fresh symbol of type tau.

type Equations
 $\mathbf{def} (=) : \mathbf{Real} \rightarrow \mathbf{Real} \rightarrow \mathbf{Equations}$

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Type Soundness

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Proposition 3 (Symbolic Lifting Preserves Types). *If $\Gamma \vdash_L e \rightsquigarrow e' : \tau$ then e' is well typed in Γ at type τ .*

Proposition 4 (Cast Insertion Preserves Types). *If $\Gamma \vdash_C e \rightsquigarrow e' : \tau$ then $\Gamma \vdash e' : \tau$.*

Lemma 3 (Progress). *If $\vdash e : \tau$ then $e \in \mathbf{Values}$, or for all S there exists S' and e' such that $e \mid S \longrightarrow e' \mid S'$, or $e = \mathbf{error}$.*

Lemma 7 (Preservation). *If $\Gamma \vdash e : \tau$ and $e \mid S \longrightarrow e' \mid S'$ then $\Gamma \vdash e' : \tau$.*

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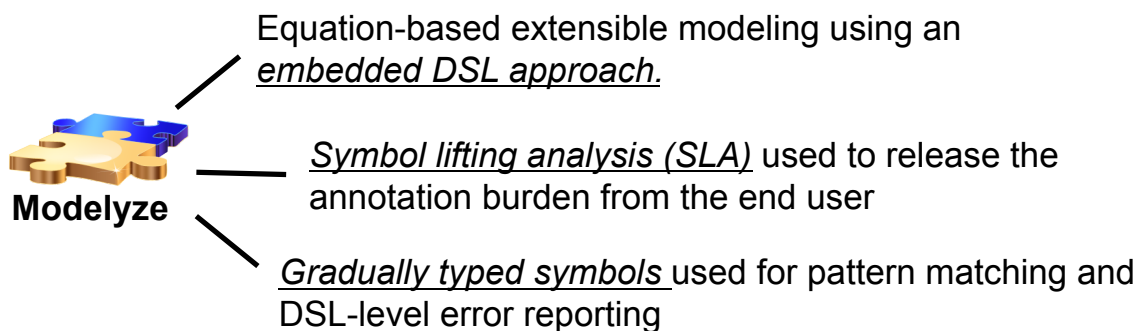


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Conclusions



Thanks for listening!

See journal preprint:

David Broman and Jeremy G. Siek. **Modelyze: a Gradually Typed Host Language for Embedding Equation-Based Modeling Languages**", Preprint, Submitted to Science of Computer Programming. Available as Tech. Report UCB/EECS-2012-173, University of California, Berkeley, June, 2012.

Open source implementation: <http://www.eecs.berkeley.edu/~broman/>

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